1. **The displacement of an object varies with time according to the equation, $y = \left(\frac{3}{8}t^2 - 3t + 5\right)$ m. Find the instataneous velocity at $t = 4$s.**

(A) 45 ms^{-1} (B) 12 ms^{-1} (C) 3 ms^{-1} (D) 0 ms^{-1}

$y = \frac{3}{8}t^2 - 3t + 5$

$\therefore v = \frac{dy}{dt} = \frac{3}{8}(2t) - 3 + 0 = \frac{3t}{4} - 3$

$\boxed{v = \frac{3t}{4} - 3}$

$\therefore t = 4s \Rightarrow v = \frac{3(4)}{4} - 3 = 0\ m/s$

2. **Acceleration of a particle is given by $\vec{a} = (2t + 5)\hat{i}$ ms^{-2}. Calculate the velocity of particle after 5*s*, if it starts from rest.**

(A) $25\hat{i}\,\frac{m}{s}$ (B) $50\hat{i}\,\frac{m}{s}$ (C) $75\hat{i}\,\frac{m}{s}$ (D) $100\hat{i}\,\frac{m}{s}$

$\therefore a = 2t + 5$ $\quad (\because v = \int a\,dt)$

$\therefore v = \int_0^5 a\,dt$

$\therefore v = \int_0^5 (2t+5)\,dt$

$= \left[\frac{2t^2}{2} + 5t\right]_0^5$

$= \left[t^2 + 5t\right]_0^5$

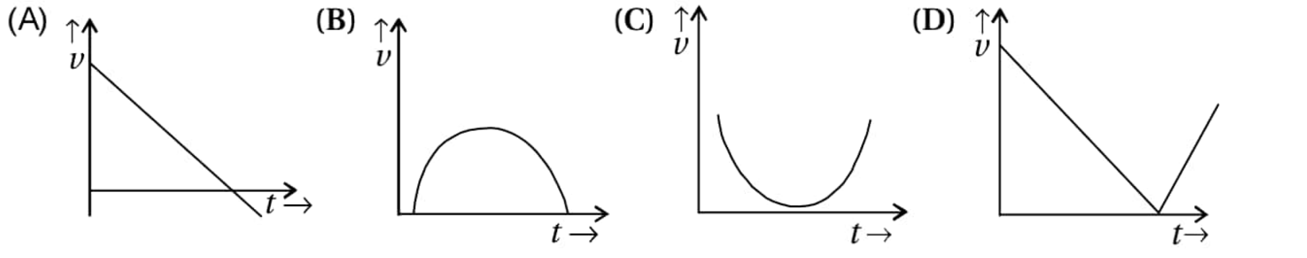
$v = \left[t^2 + 5t\right]_0^5$

$= \left[(5)^2 + 5(5)\right] - (0+0)$

$= 25 + 25$

$= 50\,\hat{i}\,\frac{m}{s}$.

3. **A particle is thrown in vertically upward direction, the correct graph of speed (v) $\rightarrow$ time (t) is ......**



(A) *a* (B) *d* (C) *c* (D) *b*

4 Velocity of a particle is given by $v = (3t^2 + 2t)\ \frac{m}{s}$. Find its average velocity between $t = 0$ to $t = 3$s and also find its acceleration at $t = 3$s. Motion of the particle is in one dimension.

(A) $11\frac{m}{s}, 10\frac{m}{s^2}$ (B) $12\frac{m}{s}, 20\frac{m}{s^2}$ (C) $11\frac{m}{s}, 20\frac{m}{s^2}$ (D) None of the given

$v = (3t^2 + 2t)$
Integrate it
$\therefore x = \int v\, dt$
$= \int (3t^2 + 2t)\, dt$
$= \left[\frac{3t^3}{3} + \frac{2t^2}{2}\right]$
$\boxed{x = t^3 + t^2}$

$\langle \vec{v} \rangle = \frac{x_2 - x_1}{t_2 - t_1}$
$= \frac{x(3) - x(0)}{3 - 0}$
$= \frac{[3^3 + 3^2] - [0]}{3}$
$= \frac{36}{3}$
$\boxed{\langle \vec{v} \rangle = 12}$ m/s.

$a = \frac{dv}{dt} = \frac{d}{dt}[3t^2 + 2t]$
$= 3 \times 2t + 2$
$\boxed{a = 6t + 2}$

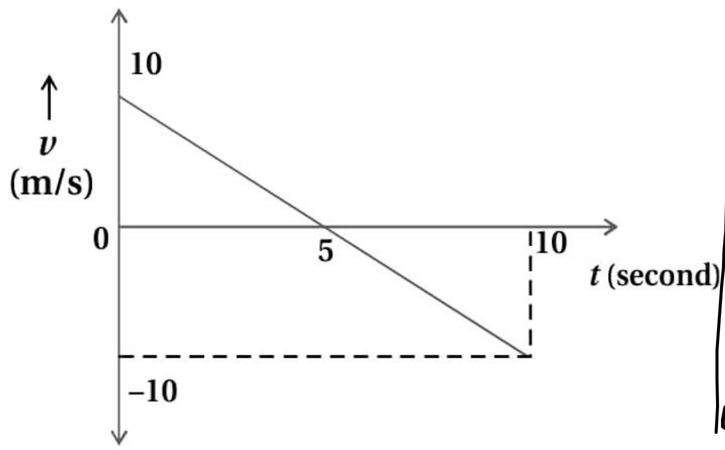
$t = 3 \Rightarrow$
$a = 6(3) + 2$
$\boxed{a = 20\ \frac{m}{s^2}}$

5 If velocity (in ms^{-1}) varies with time as $V = 5t$, find the distance travelled by the particle in time interval of $t = 2$ s to $t = 4$ s.

(A) 24 m (B) 30 m (C) 10 m (D) 20 m

$x = \int v\, dt$
$\therefore x = \int_2^4 5t\, dt$
$= \left[\frac{5t^2}{2}\right]_2^4 = \left[\frac{5(4)^2}{2} - \frac{5(2)^2}{2}\right] = \frac{5}{2}[16 - 4] = \frac{5 \times 12}{2}$ $\boxed{= 30\,m = x}$

6. Figure shows the graph of velocity versus time for a moving particle. The total distance travelled by the particle in time interval from 0 to 10 s is ......



Note :-
Area Under $\vec{v} \to t$ gives $\vec{x}$ (displacement.)

speed $\to t \Rightarrow$ path length (Total distance)

(A) –50 m (B) 50 m (C) 0 m (D) 100 m

Here: total displacement $= \left[\frac{1}{2} \times 10 \times 5 - \frac{1}{2} 10 \times 5\right] = 0$ m

total Distance $= \left[\frac{1}{2} \times 10 \times 5 + \frac{1}{2} \times 10 \times 5\right] = 25 + 25 = 50$ m

7. The position of a particle moving along a straight line is given by $x = 2 - 5t + t^3$. The acceleration of the particle at $t = 2$ sec. is ...... Here x is in meter.

(A) $12\ m/s^2$ (B) $8\ m/s^2$ (C) $7\ m/s^2$ (D) None of these

$x = 2 - 5t + t^3$

$a(x) = ?$

Note :- $a = \frac{d^2x}{dt^2}$

$v = \frac{dx}{dt} = 0 - 5 + 3t^2 = 3t^2 - 5$

$\therefore a = \frac{dv}{dt} = \frac{d}{dt}[3t^2 - 5] = \boxed{6t = a}$

$\because t = 2 \Rightarrow \boxed{a = 12\ \frac{m}{s^2}}$

8. The displacement of particle with respect to time is $s = 3t^3 - 7t^2 + 5t + 8$ where s is in m and t is in s, then acceleration of particle at t = 1 s is ......

(A) $14\ ms^{-2}$ (B) $18\ ms^{-2}$ (C) $32\ ms^{-2}$ (D) zero

Ans (C) $32\ ms^{-2}$

9. Displacement of particle changes with respect to time according to equation $x = ae^{-\alpha t} + be^{\beta t}$ where a, b, α and β are positive constants, then velocity of particle is ......

(A) independent of α and β.

(B) will be zero if $\alpha = \beta$.

(C) will decrease with respect to time.

(D) will increase with respect to time.

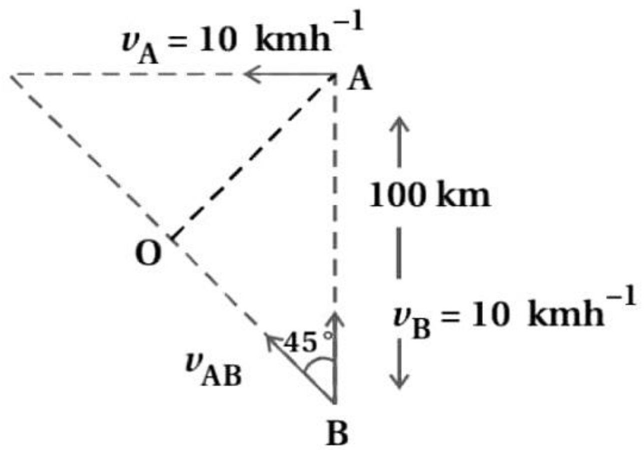
$x = ae^{-\alpha t} + be^{\beta t}$

$v = \frac{dx}{dt} = [a e^{-\alpha t} \times (-\alpha) + b e^{\beta t}(\beta)] = [b\beta e^{\beta t} - \alpha a e^{-\alpha t}] = v$

As $t \uparrow$ increase smaller & smaller So, v will increases.

10. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A, is moving Northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest, is :

(A) 0 hr (B) 5 hr (C) $5\sqrt{2}$ hr (D) $10\sqrt{2}$ hr



11. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$ where β and n are constants and x is the position of the particle. The acceleraion of the particle as a function of x, is given by :

(A) $-2n\beta^2 x^{-4n-1}$ (B) $-2\beta^2 x^{-2n+1}$ (C) $-2n\beta^2 e^{-4n+1}$ (D) $-2n\beta^2 x^{-2n-1}$

$\therefore v = \beta x^{-2n}$

$\frac{dv}{dx} = \frac{d}{dx}[\beta x^{-2n}] = \beta \frac{d}{dx}[x^{-2n}] = \beta(-2n)x^{-2n-1} = \boxed{-2n\beta x^{-2n-1} = v}$

$\therefore a = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dv}{dx} \times \frac{dx}{dt} = \boxed{\frac{dv}{dx}(v) = a}$

$\therefore a = v\frac{dv}{dx}$

$a = (\beta x^{-2n})(-2n\beta x^{-2n-1})$

$\boxed{a = -2n\beta^2 x^{-4n-1}}$

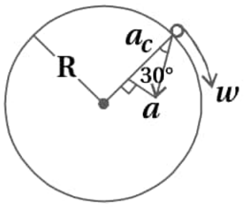
12. In the given figure $a = 15\ \text{m/s}^2$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5$ m at a given instant of time. The speed of the particle is ......

(A) 5.7 m/s

(B) 6.2 m/s

(C) 4.5 m/s

(D) 5.0 m/s



$a\cos 30° = a_c$

$\therefore a\frac{\sqrt{3}}{2} = a_c$

$\therefore \boxed{\frac{15\sqrt{3}}{2} = a_c}$

$F_c = \frac{mv^2}{R}$

$\therefore \cancel{m}a_c = \frac{\cancel{m}v^2}{R}$

$\therefore \left[a_c = \frac{v^2}{R}\right]$

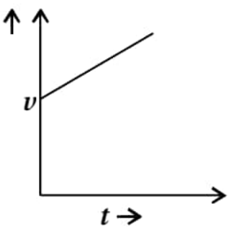
$v = \sqrt{a_c R}$

$= \sqrt{15\frac{\sqrt{3}}{2} \times 2.5}$

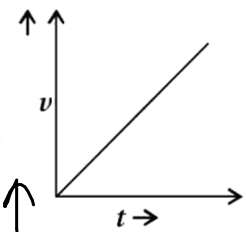
$\boxed{v = 5.7\ m/s}$

13. An object starts uniformly accelerated motion from rest. Which of the following $v \to t$ graph is correct for it ?

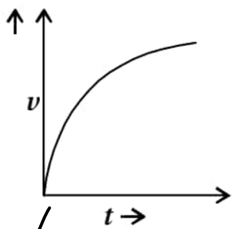
(A)



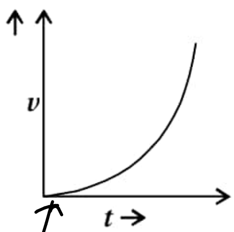
(B)



(C)



(D)

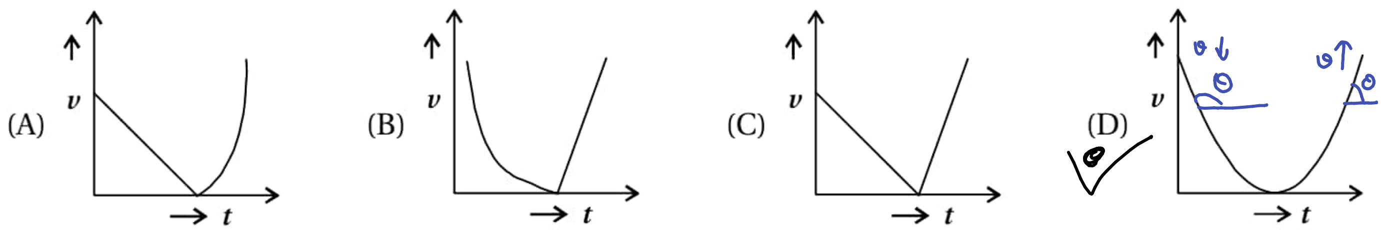


$a = 0$

($a \to$ uniform) ✓

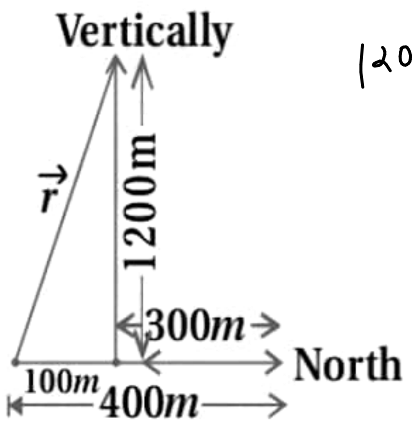
In starting $v = 0 \Rightarrow a = 0$.

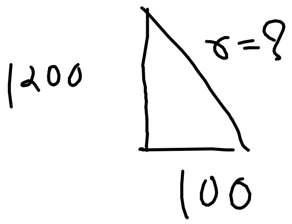
14. A ball is thrown upwards. Which of the following $x \rightarrow t$ graph is correct ? Neglect the air resistance.



15. Plane travels 400 m towards north then 300 m towards south and 1200 m vertically upwards, then find resultant displacement ....

(A) 1200 m (B) 1300 m (C) 1400 m (D) 1500 m





$r = \sqrt{(1200)^2 + (100)^2}$

$= \sqrt{1440000 + 10000}$

$= \sqrt{1450000}$

$= 1204$ m

$\boxed{r = 1200\ m}$

16. Rate of decrease of velocity of an object moving with 6.25 m/s is $\frac{dv}{dt} = -2.5\sqrt{v}$. Where v is instantaneous speed. Time taken by object to come to rest is ....

(A) 1s (B) 2s (C) 4s (D) 8s

$\frac{dV}{dt} = 6.25$ m/s.

$\therefore \frac{dV}{dt} = -2.5\sqrt{v}$.

$\therefore \frac{dV}{\sqrt{v}} = -2.5\ dt$

$\therefore \int_{6.25}^{0} \frac{1}{\sqrt{v}}\, dv = -2.5 \int_{0}^{t} dt$

$\therefore \left[-2\sqrt{v}\right]_{6.25}^{0} = -2.5t$

$\therefore \left[2\sqrt{0} - 2\sqrt{6.25}\right] = 2.5t$

$\therefore 2.5t = 2 \times 2.5$

$\boxed{t = 2s}$

time never -ve.